Optical precursors with finite rise and fall time

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We report results of both theoretical and experimental studies of optical precursors generated from a square-modulated probe laser pulse having a finite rise and fall time and propagating through a cold atomic ensemble, under the conditions of either a two-level Lorentz absorber system or a three-level system with electromagnetically induced transparency (EIT). Because of the finiteness of the rise (and fall) time, the precursor signal is observed to decrease with increasing optical depth ($\alpha_0L$). We find that the precursor can experience little absorption even at high optical depth if the rise (and fall) time is sufficiently short. At an optical depth of $\alpha_0L=42$, the normalized precursor peak intensity is observed to increase from 9% to 27% when the rise (and fall) time is shortened from 7 to 3 ns. Meanwhile, we reaffirm that there is no violation of Einstein’s causality principle in light propagation through both slow and fast light media. In the EIT system with high optical depth, the main field propagates with a subluminal group velocity and it is separated from the precursor. In the two-level system, the effect of negative group velocity in the anomalous dispersion regime is observed, but we detect no advancement in the rising edge of the precursors. In both cases, the leading edges of the precursors show no detectable delay to that through vacuum.

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I. INTRODUCTION

In 1914, Sommerfeld and Brillouin studied theoretically the propagation of a step-modulated optical pulse through a linear dispersive medium and found the remarkable fact that the wave front at the rising edge always travels at the speed of light in vacuum $c$ [1, 2]. This wave front, in the form of a transient wave, is now known as the optical precursor. Since then, theoretical [3–9] and experimental [10–14] confirmation of the existence of optical precursors has been a subject of fundamental interest because of its connection to Einstein’s causality principle [15]. But until recently, some researchers still questioned whether precursors had indeed been observed in previous experiments [16–18]. Very recently, we reported the observation of precursors that are clearly separated from the main pulse when a long square laser pulse passes through a cold atomic medium with electromagnetically induced transparency (EIT) [19, 20] and at high optical depth [21]. Our results also confirm that in a two-level narrow-line Lorentz absorber, when the resonance main field is completely absorbed, the left-over standing-alone transient spikes at the rising and falling edges are indeed the precursors [21].

Theoretically, the wave front of the precursor at the rising edge of an ideal step-modulated optical pulse is expected to propagate with zero absorption at all optical depths [7–9]. However, our experiment showed that, for a real step pulse with finite rise time, the precursor signal decreases as we increase the optical depth [21]. Oughstun has theoretically investigated this finite rise-time effects on the precursor field formation [22]. This propagation loss of optical precursors has also been studied indirectly in broadband femtosecond pulses [23, 24]. In this paper, we report the direct measurement of the dependence of propagation loss of optical precursors on rise (fall) time. Our results show that the transmission loss of precursors can be significantly reduced by shortening the rise and fall time. At a high optical depth of about 42, the precursor peak transmission is observed to increase from 9% to 27% when the rise (fall) time is shortened from 7 to 3 ns. The results suggest that precursors may have possible applications in under-water optical communication and biomedical imaging [25].

The paper is organized as follows. In Sec. II, we present a simple theoretical model that we use to describe optical precursors generated from a step pulse with finite rise time. We then show in Sec. III the experimental results obtained at different optical depths and with rise (fall) time of 7 and 3 ns. In Sec. IV, we show that slow and fast light effects can be observed in the main pulse and in both cases the main pulse is well behind the precursor, reaffirming that there is no violation to Einstein’s causality principle. Finally, we draw a conclusion in Sec. V.

II. THEORY

The schematics of a three-level EIT system is shown in Fig. 1. The atoms are prepared in the ground state $|1\rangle$. In presence of a strong coupling laser ($\omega_c$), on resonance with the transition $|2\rangle \rightarrow |3\rangle$, the medium becomes transparent at the probe laser ($\omega_p$) transition $|1\rangle \rightarrow |3\rangle$ [19, 20]. The steep linear dispersion in the EIT narrow transparency window results in a slow group velocity [26]. Obviously, when we turn off the coupling laser, it becomes a two-level system where the probe field sees on-resonance absorption and anomalous dispersion with

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negative group velocity. Therefore, the system allows us to study optical precursors in both slow and fast light media by controlling the coupling laser in a single experimental apparatus.

We use the formal notation \( \frac{1}{2}E(z, t)e^{i[k_0z−\omega t]} + \text{c.c.} \) to describe the probe laser field, where \( E(z, t) \) is the complex envelope. \( \omega = \omega_\text{0} \) and \( k_0 = \omega_\text{0}/c \) are the probe carrier angular frequency and wave number in vacuum. For linear propagation of a weak probe pulse, the output optical field envelope can be obtained from the integral

\[
E(t) = \frac{1}{2\pi} \int E_0(\omega)e^{i(\Delta k(\omega)−\omega t)}d\omega, \tag{1}
\]

where \( E_0(\omega) = \int E_0(t)e^{i\omega t}d\omega \) is the spectrum of the initial input pulse envelop at \( z = 0 \), \( L \) is the medium length, and \( \Delta k = k_0\sqrt{1+\chi}−k_0 \approx k_0\chi/2 \). With \( \omega \) denoted as frequency detuning from the carrier \( \omega_\text{0} \), the EIT linear susceptibility (complex) is given as

\[
\chi = \frac{\alpha_0}{\Omega_\text{c}^2−4(\omega+i\gamma_{12})(\omega+i\gamma_{13})}, \tag{2}
\]

where \( \alpha_0 \) is the on-resonance absorption coefficient when the coupling laser is off, and \( \Omega_\text{c} \) is the coupling laser Rabi frequency. \( \gamma_{ij} \) is the atomic dephasing rate between \( |i\rangle \) and \( |j\rangle \). In an ideal EIT system, the ground state dephasing rate is typically very small, i.e., \( \gamma_{12} \approx 0 \). When the coupling laser is switched off (\( \Omega_\text{c} = 0 \)), Eq. (2) reduces to that of a two-level system.

An ideal step-modulated input probe laser pulse can be written as \( E_\text{0}(\pm t) = E_\text{0}\Theta(\pm t) \), where \( \Theta(t) \) is the Heaviside function and the sign (+) represents the step-on (−off) pulse with a rising (falling) edge. There have been many theoretical studies on propagation of a step-modulated pulse through a resonant two-level Lorentz absorber \([5, 9]\) or an EIT system \([7, 8]\). At high optical depth, both yield nearly identical Sommerfeld-Brillouin precursors envelopes \([8, 21]\):

\[
E_{SB}(t) \approx \pm E_\text{0}J_0\sqrt{2\alpha_0L/\gamma_{13}}e^{−\gamma_{13}\tau}, \tag{3}
\]

where \( \tau = t−L/c \), \( J_0 \) is the 0-order first kind Bessel function. A more detailed derivation using uniform asymptotic expansion can be found in Refs. \([4, 9]\).

In this paper, we study the optical pulse whose carrier frequency is on resonance at the transition \( |1\rangle \rightarrow |3\rangle \). Therefore in the two-level system when the coupling laser is off, the carrier frequency \( (\omega_p) \) is on absorption band and its behavior has been studied theoretically using the uniform asymptotic analysis \([6]\). At high optical depth, this main field is absorbed and only the precursor field is left over. For the EIT system, the main field within the narrow transparency window, traveling with a slow light group delay of \( \tau_g \approx 2\alpha_0L/\gamma_{13}/|\Omega_\text{c}|^2 \), can be approximated as \([21]\):

\[
E_{M±}(t) \approx \frac{E_\text{0}}{2}(1±\text{erf}[\frac{\sqrt{2\alpha_0L\tau/\gamma}}{2\sqrt{2}\tau_g}])e^{i\Delta k(0)L}, \tag{4}
\]

where \( \text{erf} \) is the error function. The total field envelop is \( E_{±}(t) = E_{SB}(t) + E_{M}(t) \).

To study the finite rise-time effects, Oughstun suggested to use a hyperbolic tangent function to describe a real step pulse \([22]\). However, the theoretical analysis of such a hyperbolic tangent function based on the asymptotic approach is complicated \([22]\). Here we take a simple approach and model a realistic step pulse by turning on (off) the field amplitude linearly with a finite rise (fall) time of \( \Delta t \). Mathematically, this can be done by convoluting the ideal step pulse and a quare function having a time duration of \( \Delta t \):

\[
\tilde{E}_\text{0}(t) = \frac{1}{\Delta t}E_\text{0}(t) * \Pi(t, \Delta t) = \frac{1}{\Delta t}\int_0^{\Delta t} E_\text{0}(t−t′)\Pi(t′, \Delta t)dt′ = \frac{1}{\Delta t}\int_0^{\Delta t} E_\text{0}(t−t′)dt′. \tag{5}
\]

The unit square function is defined as \( \Pi(t, \Delta t) = 1 \) for \( t \in [0, \Delta t] \) and otherwise zero. Then the output precursor field becomes

\[
\tilde{E}_{SB±}(t) = \frac{1}{\Delta t}\int_0^{\Delta t} E_{SB±}(t−t′)dt′. \tag{6}
\]

Equation (6) gives a clear picture in time domain: the averaging effect within the rise (fall) time window reduces the peak values of the precursors. There are two important characteristic times that determine the shape of precursors. At first, the duration of first peak of the precursor is determined by the Bessel function in Eq. (3): \( \tau_s = \pi^2/(2\alpha_0L/\gamma_{13}) \) \([21, 27]\). The second is the precursor (intensity) decay time constant \( \tau_d = 1/(2\gamma_{13}) \) determined by the atomic natural linewidth. Therefore, in order to detect the optical precursor with high visibility, it requires that the finite rise (fall) time is shorter than the both characteristic times, i.e., \( \Delta t < \{\tau_s, \tau_d\} \).

In frequency domain, the effect of finite rise (fall) time works as a low-pass filter with a bandwidth determined by \( 1/\Delta t \):

\[
\Phi(\omega) = \frac{1}{\Delta t}\int_0^{\Delta t} \Pi(t, \Delta t)e^{−i\omega t}dt = \frac{\text{sinc}(\omega\Delta t/2)e^{−i\omega\Delta t/2}}{i\omega\Delta t/2}, \tag{7}
\]
where \( \text{sinc}(x) = \sin(x)/x \). For high optical depth, the Sommerfeld and Brillouin precursor frequency saddle points move far away from the atomic resonance [7, 9] and are attenuated by the filter effect caused by the finite rise and fall time. We notice that, the detection bandwidth of the equipment has also similar effects on reducing the measured precursor transients [12, 28]. As described in next section, to avoid the effect from the finite detection bandwidth, we use a detector with much higher response speed than the rising (falling) edge of the square pulse.

### III. EXPERIMENT

The experimental system used in this work is similar to the one reported in Ref. [21]. We work with cold atoms in a two-dimensional (2D) \(^{85}\text{Rb}\) magneto-optical trap (MOT) with a longitudinal length \( L = 1.5 \text{ cm} \) and a temperature of about 100 \( \mu \text{K} \). The energy level configuration is taken as the following in \(^{85}\text{Rb}\) D1 line (795 nm): \( |1\rangle = |5S_{1/2}, F = 2\rangle \), \( |2\rangle = |5S_{1/2}, F = 3\rangle \), and \( |3\rangle = |5P_{1/2}, F = 3\rangle \). The dephasing rates are \( \gamma_{13} = 2\pi \times 3 \text{ MHz} \) and \( \gamma_{12} = 0.01\gamma_{13} \). To ensure that linear propagation effect is studied, we keep the intensity of the probe laser, locked to the resonance of the \( |1\rangle \rightarrow |3\rangle \) transition, sufficiently low that the atomic population remains primarily in the state \( |1\rangle \). The coupling and probe beams cross each other at the MOT with a 2 degree angular separation so that they can be separated when they reach the detector (PMT, Hamamatsu, H6780-20, 0.78 ns rise time). The data is recorded using a 1 GHz realtime digital oscilloscope (Tektronix, TDS684B) and averaged over 30 traces. To study optical precursor transient response at both the rising and falling edges in a single measurement, we used a probe laser beam that is modulated by a long square pulse with a length of 2 \( \mu \text{s} \).

We first measure the optical precursors from a weak long-square pulse propagating through an EIT system with a finite rise and fall time of 7 ns. \( \Omega_c = 4\gamma_{13}, \alpha_0L = 42 \). The red solid lines are numerical simulation using FFT. (b) and (c) are zoomed views around the rising and falling edges.

We next set the coupling beam phase to zero and keep the intensity of the probe laser, locked to the resonance of the \( |1\rangle \rightarrow |3\rangle \) transition, sufficiently low that the atomic population remains primarily in the state \( |1\rangle \). The rising and falling edges in a single measurement. We use a probe laser beam that is modulated by a long square pulse with a length of 2 \( \mu \text{s} \).

We first measure the optical precursors from a weak square pulse with a rise (and fall) time of \( \Delta t = 7 \text{ ns} \), gen-
Transmission with a shorter rise (and fall) time of $\Delta t = 3 \text{ ns}$. $\Omega_c = 0$, $\alpha_0 L = 42$. The red solid lines are numerical simulation using FFT. (b) and (c) are zoomed views around the rising and falling edges.

FIG. 5: (color online). Observation of optical precursors from a weak long-square pulse propagating through a two-level system with a finite rise and fall time of 3 ns. $\Omega_c = 0$, $\alpha_0 L = 42$. The red solid lines are numerical simulation using FFT. (b) and (c) are zoomed views around the rising and falling edges.

We then repeat the measurements using a square pulse with a shorter rise (and fall) time of $\Delta t = 3 \text{ ns}$, generated using an electro-optical modulator (EOM, EOSpace) driven by the same digital delay/pulse generator. The results obtained from the EIT and two-level systems are shown in Figs. 4 and 5, respectively. The data are qualitative the same as those shown in Figs. 2 and 3. However, by shortening the rise and fall time from 7 to 3 ns, the normalized peak intensity of the precursor is observed to increase from 9% to about 27%.

By varying the optical depth ($\alpha_0 L$) from 0 up to 45, we measure the attenuation of optical precursors for $\Delta t = 3$ and 7 ns. Figure 6 shows normalized transient peak intensity at the rising edge as a function of optical depth for both the EIT and two-level systems. As expected, faster rise time results in higher transient peak intensity. We also show in Fig. 6 the theoretical curves calculated for 1 ns rise, which suggest that still higher precursor intensity can be achieved by using a faster light modulator. These experimental results suggest that precursors may have potential applications in optical communication through lossy medium.

IV. SLOW AND FAST LIGHT

Equation (4) in Sec. II shows that in the EIT slow light medium, the main field is delayed by the group delay time $\tau_g$. This makes physical sense for slow light. However, if one extends this idea to the case of negative
group velocity in a fast light medium, it would seem that
the main field may advance ahead of the rising edge of
the precursor, leading to violation of Einstein’s causality
principle. In this section, we show experimentally that
this does not happen and Einstein’s causality is upheld
in both slow and fast light media.

First, we consider the EIT slow light case. Figure 7
shows the propagation of a Gaussian as well as a step
pulses through an EIT medium having an optical depth of
\( \alpha_0 L = 30 \). In the case of Gaussian pulse, we measured
a group delay of about 200 ns with no attenuation and
distortion [Fig. 7(a)]. This pulse delay is consistent with
the observation that the delayed main field that turns
on smoothly after 200 ns [Fig. 7(b)]. Meanwhile, the
leading edge of the precursor shows no detectable delay
to the step pulse.

In the case of a two-level Lorentz absorber system, the
on-resonance group velocity becomes negative and the
main field experiences absorption. One may argue that
Einstein’s causality can not be violated in the propaga-
tion of step pulse because the main field is essentially
completely absorbed at high optical depth as shown in
Figs. 3 and 5. But this may not be true for low optical
depth. In Fig. 8, we show the propagation of both Gauss-
ian and step pulses at a low optical depth \( \alpha_0 L = 2.3 \)
where substantial main field (more than 10% of the in-
put pulse) is present. In this case, we observe a signif-
ificant peak advancement of 50 ns in the Gaussian pulse
propagation [Fig. 8(a)] with obvious attenuation and dis-
tortion. However, for the step pulse, we observe no ad-
vance of the rising edge [Fig. 8(b)]. Since information
is encoded at a non-analytic wave front [29–31], our
experiment confirms that there is no violation of Ein-
stein’s Causality principle in light propagation through
fast light medium and the information velocity is differ-
ent from the group velocity.

In our experiment, the long Gaussian pulses have nar-
row linewidths compared to the medium dispersion, thus
their peaks propagate with the group velocity. As the
pulse temporal length becomes short and the pulse spec-
trum is wide, it reduces to the energy velocity descrip-
tion [32].

In Fig. 9, we show the precursor peak duration and
group delay as a function of the optical depth for a square
pulse propagating in the EIT medium. The peak dura-
tion is measured from the first peak of the precursors,
which can be fitted well by the curve calculated from
\( \tau_s = \pi^2/(2\alpha_0 L\gamma_{13}) \). The group delay is measured from
the rising edge [Fig. 9(b): circle points] and falling edge
[Fig. 9(b): square points] to the point where the main
field transmission is 25% [see Eq. (4)]. Both set of data
agree very well with the theoretical curves obtained di-
rectly from the EIT linear dispersion. The above results
show clearly that precursors do not propagate with the
group velocity of the main field.

V. CONCLUSION

In summary, we have studied, in both theory and
experiment, optical precursors generated from a low-
intensity square-modulated on-resonance laser pulse hav-
ing a long pulse duration and a finite rise (and fall) time.
Our experimental system allows us to switch between
the EIT system and two-level Lorentz absorber system
by turning on and off the coupling laser. This enables
us to modify the medium dispersion parameters over a
wide range. Additionally the optical depth can be varied
from 0 up to 45. Our experiments show that the optical
precursors in both the EIT and two-level systems can
experience little absorption even at high optical depth if
the rise (and fall) time is sufficiently short. For example,
at an optical depth \( \alpha_0 L = 42 \), the normalized precursor
peak intensity is observed to increase from 9% to 27% when
the rise (and fall) time is shortened from 7 to 3 ns. In time domain, shortening the rise (fall) time effec-
tively reduces the averaging time [Eq. (6)]. In frequency
domain, a step pulse with a shorter rise (fall) time con-
tains a wider spectrum and more spectral components
fall outside of the absorption band [Eq. (7)]. As a re-
result, the precursors get absorbed less. Our simple the-
toretical analysis of the finite rise (fall) time effect agree well with the experimental observation in a wide parameter range. Our results suggest that precursors may find potential applications in optical communication through lossy medium.

We also reaffirm that Einstein’s causality principle is upheld in light propagation through both slow and fast light media. In the EIT slow light case, the main field of a step pulse is delayed from the precursor, consistent with the pulse delay measurement we performed using a Gaussian pulse. In the two-level fast light case, we measured a negative pulse delay of about -50 ns for a Gaussian pulse, but no advancement in the rising edge of the precursor of the step pulse is detected.

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